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Magnetic Properties of Weakly Doped Antiferromagnets from the Lagrangian t–J Model

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From the Lagrangian formalism for the t-J model previously developed, renormalized magnetic properties in weakly doped antiferromagnets can be evaluated. The renormalization effects essentially appear because of the interaction of particle-hole with the spin wave. For small concentration of holes the self-energies are computed. Taking an approximate form for the particle spectral function, the cuasiparticle peak and the incoherent continuum region are analyzed in order to evaluate the softening and the damping in the spin excitations of antiferromagnets weakly doped. The results can be confronted with previous one obtained by means of the Hamiltonian t-J model in the slave-fermion Schwinger boson representation.

KEY WORDS: Antiferromagnetism; t-J model; Lagrangian formalism; Hubbard operators.

1. INTRODUCTION

The actual interest in the study of the magnetic properties in some weakly doped antiferromagnets is due to their connection with high temperature superconductivity. Among others results, the experiments have shown important softening and damping in the spin excitations (Hayden *et al.*, 1991, 1996) as well as an increase in the transversal susceptibility (Nakano *et al.*, 1994; Ohsugi *et al.*, 1997; Song *et al.*, 1993) when interaction between holes and spin waves are considered. The undoped configuration is an antiferromagnetic insulator. Doping produces holes (Heuser *et al.*, 1998; Mathur *et al.*, 1998; Schröder *et al.*, 1998; Stockert *et al.*, 1998; Sullow *et al.*, 1999a,b; Tjeng *et al.*, 1997) and the long-range antiferromagnetic order rapidly disappears at low doping, and superconductivity arises upon further doping. The motion of holes strongly interacting with the spin array generates a renormalization of the magnetic properties. This well-known effect is usually studied from the t–J Hamiltonian model (Igarashi and Fulde, 1992;

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Izyumov, 1997; Martinez and Horsch, 1991; Pimentel *et al.*, 1999). In the current literarure, the t–J Hamiltonian model is written in the slave-fermion representation for the case of an antiferromagnetic matrix. Considering the model in a Schwinger boson representation, hole motion is treated within a self-consistent Born approximation (Kane *et al.*, 1990; Liu and Manousakis, 1992; Schmitt-Rink *et al.*, 1988; Sullow *et al.*, 1999a,b); that is, the self-consistent equation for the hole self-energy is solved numerically and so, by using the Dyson equation, the hole dressed propagator can be found. This method has been applied considering different approaches for the propagation of a single hole in a two dimensional antiferromagnet. Next the study of hole motion has been extended for a finite concentration of holes (Igarashi and Fulde, 1992; Kyung and Mukhim, 1997; Plakida *et al.*, 1994).

In the present work starting from the renormalized first-order Lagrangian formalism for the t–J model previously developed (Foussats *et al.*, 2000 a,b, 2002) magnetic properties for weakly doped antiferromagnets are evaluated. The results are in agreement with available experimental data and constitute a strong proof on the validity of our Lagrangian model.

The paper is organized as follows. In Section 2, the propagators and vertices that result from the Lagrangian model are analyzed. In Section 3, the different contributions to the self-energies for both hole and spin-wave fields are computed in order to obtain dressed propagators. In section 4, magnetic properties in antiferromagnets weakly doped are evaluated. In particular the softening and the damping in the spin excitations are analyzed.

2. LAGRANGIAN, PROPAGATORS AND VERTICES FOR ANTIFERROMAGNETIC CONFIGURATION

The starting point is to consider the renormalized Lagrangian formalism developed by Foussats *et al.* (2002). Let us assume that we are close to an undoped regime where the system is an antiferromagnetic insulator. Under this condition there is a small number of holes and it can be assumed that the hole density $\rho_i = \langle \rho_i \rangle =$ constant. For a given value of the chemical potential μ, the constant value of the hole density must be determined by consistency.

As it is usual in the antiferromagnetic configuration a rotation of spins on the second sublattice by 180° about the S_1 axis must be performed (Manousakis, 1991),

$$S_{j1} \to S_{j1},\ \ S_{j2} \to -S_{j2},\ \ S_{j3} \to -S_{j3},\ \ \Psi_{j\sigma} \to \Psi_{j\bar{\sigma}}, \tag{2.1}$$

where $\sigma \to \bar{\sigma}$ implies $\pm \to \mp$.

From now on, the system fluctuating around an antiferromagnetic state ($J_{ij} <$ 0) is assumed. In such conditions, the components of the real vector field S are close to be the spin variables, and so the vector S is written

$$\mathbf{S} = (0, 0, s') + (\widetilde{S}_1, \widetilde{S}_2, \widetilde{S}_3) \tag{2.2}$$

where $\widetilde{S}_1$, $\widetilde{S}_2$, $\widetilde{S}_3$ are the fluctuations. In order to simplify notation hereafter the tilde over the fluctuations is omitted.

This canonical tranformation (2.1) changes the antiferromagnetic configuration into a ferromagnetic one with all spins up, and so it is not necessary to distinguish between sublattices. However the effective Lagrangian is not invariant under such transformation, because the noninvariance of the t–J Hamiltonian.

Consequently, by following Foussats (2002), the effective Lagrangian for the t–J model is written in terms of four bosonic-field components (S_1, S_2, S_3, λ) and two fermionic-field components (Ψ^*_-, Ψ_-).

Once the transformation to the Euclidean space is carried out, the effective Lagrangian in terms of the fluctuations (2.2) takes the form

$$\begin{aligned}
L^E_{\text{eff}} = {} & \frac{i}{2s}(1-\rho)\sum_i \frac{S_{i1}\dot{S}_{i2} - S_{i2}\dot{S}_{i1}}{s+s'}\left[1 + \sum_{n=1}(-1)^n\left(\frac{S_{i3}}{s+s'}\right)^n\right] \\
& - \frac{s}{s+s'}\sum_i\left(\dot{\Psi}^*_{i-}\Psi_{i-} + \dot{\Psi}_{i-}\Psi^*_{i-}\right)\left[1 + \sum_{n=1}(-1)^n\left(\frac{S_{i3}}{s+s'}\right)^n\right] \\
& - \frac{2s\,\mu}{s+s'}\sum_i \Psi^*_{i-}\Psi_{i-}\left[1 + \sum_{n=1}(-1)^n\left(\frac{S_{i3}}{s+s'}\right)^n\right] \\
& + \frac{1}{(s+s')}\sum_{i,j} t_{ij}\Psi_{i-}\Psi^*_{j-}\,[S_{i1} - i\,S_{i2} + S_{j1} + i\,S_{j2}] \\
& + \frac{1}{(s+s')}\sum_{i,j} t_{ij}\Psi_{i-}\Psi^*_{j-}\,\left[(S_{i1} - i\,S_{i2})\left(\sum_{n=1}(-1)^n\left(\frac{S_{i3}}{s+s'}\right)^n\right)\right. \\
& \left. + \ (S_{j1} + i\,S_{j2})\left(\sum_{n=1}(-1)^n\left(\frac{S_{i3}}{s+s'}\right)^n\right)\right] \\
& - \frac{1}{8s^2}J'\sum_{i,I}\left[S_{i1}S_{(i+I)1} - S_{i2}S_{(i+I)2} - S_{i3}S_{(i+I)3} + S^2_{i1} + S^2_{i2} + S^2_{i3}\right] \\
& - 2s'\sum_i \lambda_i S_{i3} - \sum_i \lambda_i\left[S^2_{i1} + S^2_{i2} + S^2_{i3}\right],
\end{aligned} \tag{2.3}$$

The free propagator of the boson field $V^a = (S_1, S_2, S_3, \lambda)$ is an Hermitian nonsingular 4×4 dimensional matrix. From the effective Lagrangian (2.3) the expression for the free-boson propagator or spin-wave propagator in the Fourier space results

$$\mathcal{D}^{ab}_{(0)}(q,\omega_n) =$$
$$\begin{pmatrix} -\frac{J'z}{8}(s+s')^2\frac{(1-\gamma_q)}{\omega_n^2+\omega_q^2}(1+\rho)^2 & s(s+s')\frac{\omega_n}{\omega_n^2+\omega_q^2}(1+\rho) & 0 & 0 \\ -s(s+s')\frac{\omega_n}{\omega_n^2+\omega_q^2}(1+\rho) & -\frac{J'z}{8}(s+s')^2\frac{(1+\gamma_q)}{\omega_n^2+\omega_q^2}(1+\rho)^2 & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{2s'} \\ 0 & 0 & -\frac{1}{2s'} & \frac{J'z(1-\gamma_q)}{32s^2s'^2} \end{pmatrix} \tag{2.4}$$

In Eq. (2.4), q and ω_n are respectively the momentum and the Matsubara frequency of the boson field.

Moreover, in Eq. (2.4) the frequency ω_q is defined by

$$\omega_q = \frac{zJ'}{8s}(s+s')(1+\rho)\sqrt{1-\gamma_q^2}, \tag{2.5}$$

where $J' = J(1-\rho)^2 < 0$, z is the number of first nearest-neighbor sites and $z\gamma_q = \sum \hat{I} \exp(iq\cdot I)$, where I is the lattice vector.

From the above equations it can be seen that the spatial dimension of the underlying lattice and the physics depend on the parameters z and γ_q, though our Lagrangian formalism is dimensional-independent.

On the other hand the antiferromagnetic free-magnon propagator is defined by

$$D^{+-}_{(0)} = \left(D^{-+}_{(0)}\right)^* = \langle T\, S^+(\tau)S^-(0)\rangle = \frac{1}{2}\left(D^{11}_{(0)} + D^{22}_{(0)} - i\left(D^{12}_{(0)} - D^{21}_{(0)}\right)\right). \tag{2.6}$$

And from Eq. (2.4) it results

$$D^{+-}_{(0)}(q,\omega_n) = -s(s+s')(1+\rho)\left(\frac{J'z(s+s')}{8s}(1+\rho) + i\omega_n\right)\frac{1}{\omega_q^2+\omega_n^2}. \tag{2.7}$$

As well known, from the antiferromagnetic magnon propagator $D^{+-}_{(0)}(q,\omega_n)$, the magnon spectral function is defined by

$$\begin{aligned} \mathcal{A} &\equiv -\frac{1}{\pi}\lim_{\varepsilon\to 0} Im\, D^{+-}_{(0)}(\omega+i\varepsilon) \\ &= s(s+s')(1+\rho)\,[A_+(q)\,\delta(\omega-\omega_q) + A_-(q)\,\delta(\omega+\omega_q)], \end{aligned} \tag{2.8}$$

where

$$A_\pm = \frac{1}{2}\left(1 \pm \frac{1}{\sqrt{1-\gamma_q^2}}\right). \tag{2.9}$$

We note that the expression (2.8) for the antiferromagnetic magnon spectral function is the generalization to that given in Manousakis (1991) when the hole density $\rho \neq 0$. Equation (2.8) that generalizes the well-known antiferromagnetic magnon spectral function, really checks the validity of the free-propagator expression (2.7) for finite values of the hole density.

At this point it is convenient to apply the Bogoliubov–Valatin transformation for the spin variables, i.e.,

$$S^{+} = \frac{1}{\sqrt{2}}\left(\frac{S_1}{u_q + v_q} + i\,\frac{S_2}{u_q - v_q}\right), \tag{2.10a}$$

$$S^{-} = \frac{1}{\sqrt{2}}\left(\frac{S_1}{u_q + v_q} - i\,\frac{S_2}{u_q - v_q}\right), \tag{2.10b}$$

where

$$u_q = \sqrt{s(s+s')(1+\rho)\left(\frac{1+\sqrt{(1-\gamma_q^2)}}{2\sqrt{(1-\gamma_q^2)}}\right)}, \tag{2.11a}$$

$$v_q = -(sign\,\gamma_q)\sqrt{s(s+s')(1+\rho)\left(\frac{1-\sqrt{(1-\gamma_q^2)}}{2\sqrt{(1-\gamma_q^2)}}\right)}. \tag{2.11b}$$

The Bogoliubov–Valatin transformation is carried out to simplify the calculations. Once the Bogoliubov–Valatin transformation was given, the free-boson propagator components read

$$D^{+-}_{(0)}(q,\omega_n) = (D^{-+}_{(0)})^{*}(q,\omega_n) = -\frac{1}{\omega_q - i\omega_n} \tag{2.12a}$$

$$D^{++}_{(0)}(q,\omega_n) = D^{--}_{(0)}(q,\omega_n) = 0 \tag{2.12b}$$

$$D^{34}_{(0)}(q,\omega_n) = D^{43}_{(0)}(q,\omega_n) = -\frac{1}{2s'} \tag{2.12c}$$

$$D^{44}_{(0)}(q,\omega_n) = \frac{J'z}{32(ss')^2}(1-\gamma_q) \tag{2.12d}$$

On the other hand, as shown in Foussats (2002), the main problem in the antiferromagnetic configuration is to give the mechanism for the fermion propagation. From the Eq. (2.3), the bilinear fermionic part reads

$$L^{F} = \sum_{k,\nu_n} \Psi^{*}_{-}(k,\nu_n)\, G_0^{-1} \Psi_{-}(k,\nu_n), \tag{2.13}$$

where we have named

$$G_0^{-1} = -\frac{2s}{s+s'}(i\nu_n + \mu). \tag{2.14}$$

The inverse of this scalar function given by

$$G_0 = -\frac{s+s'}{2s}\frac{1}{i\nu_n+\mu}, \tag{2.15}$$

is a (nonpropagating) functional that depends only on the Matsubara frequency ν_n. Later on, the prescriptions for the propagation of the fermionic modes will be given.

Here we write only the three-leg and four-leg vertices corresponding to the interaction between spin waves, and the interaction between holes and spin waves. Looking at the Lagrangian (2.3), it can be seen that these interaction vertices respectively can be written

$$U_+ = -\frac{\sqrt{2}}{s+s'}[v_k\varepsilon_k + u_{k-q}\varepsilon_{k-q}], \tag{2.16a}$$

$$U_- = -\frac{\sqrt{2}}{s+s'}[u_k\varepsilon_k + v_{k-q}\varepsilon_{k-q}], \tag{2.16b}$$

$$U_3 = -\frac{s}{(s+s')^2}[i(v_n+v'_n)+2\mu], \tag{2.16c}$$

$$U_{+3} = \frac{\sqrt{2}}{(s+s')^2}[v_k\varepsilon_k + u_{k-q}\varepsilon_{k-q}], \tag{2.16d}$$

$$U_{-3} = \frac{\sqrt{2}}{(s+s')^2}[u_k\varepsilon_k + v_{k-q}\varepsilon_{k-q}], \tag{2.16e}$$

$$U_{33} = \frac{s}{(s+s')^3}[i(v_n+v'_n)+2\mu], \tag{2.16f}$$

and the other components vanish.

The above equation defines $\varepsilon_k = -tz\gamma_k$.

The three-boson interaction vertex F_{abc} is written

$$\begin{aligned} F_{abc} = -i\frac{(1-\rho)}{2s(s+s')^2}(\omega_2-\omega_1)\,&[(u_{q1}\,v_{q2} - u_{q2}\,v_{q1})(\delta_a^+\delta_b^+ - \delta_a^-\delta_b^-) \\ &+ (u_{q1}\,u_{q2} - v_{q1}\,v_{q2})(\delta_a^+\delta_b^- - \delta_a^-\delta_b^+)]\delta_c^3 \\ &- 2[(u_{q1}\,v_{q2} + u_{q2}\,v_{q1})(\delta_a^+\delta_b^+ + \delta_a^-\delta_b^-) \\ &+ (u_{q1}\,u_{q2} + v_{q1}\,v_{q2})(\delta_a^+\delta_b^- + \delta_a^-\delta_b^+) + \delta_a^3\delta_b^3]\delta_c^4 \\ &+ \text{ciclic rotations of indices}, \end{aligned} \tag{2.17}$$

and the analogous expression for the four-boson interaction vertex named F_{abcd} (see Foussats *et al.*, 2002).

This diagrammatics allow to compute the one-loop contribution to the perturbative development in the component S_3 of the fluctuations.

3. SELF-ENERGIES GENERATED BY THE INTERACTION BETWEEN SPIN WAVES AND HOLES—DRESSED PROPAGATORS

We begin studying the fermion self-energy. The usual way to solve the propagation of fermions (particle-hole propagation) is by means of the Dyson equation. As known, the Dyson theorem allows to compute the inverse of the corrected fermion propagator in terms of the free-fermion propagator and the self-energy. Therefore the propagator

$$G(k, \nu_n) = \left[G_0^{-1}(\nu_n) - \sum(k, \nu_n)\right]^{-1}, \tag{3.1}$$

can be evaluated in a straightforward way within the self-consistent Born approximation (SCBA) framework (see for instance Martinez and Horsch, 1991; Schmitt-Rink *et al.*, 1988).

Once an appropriate self-energy function $\sum(k, i\nu_n)$ is found, and after the analytic continuation $i\nu_n = \nu + i\delta$ is done, the propagator $G(k, \nu)$ remains well defined, and so it is possible to compute numerically the spectral function defined by $B(k, \nu) = -\frac{1}{\pi} \lim_{\delta\to 0} G(k, \nu + i\delta)$.

On the other hand, it is easy to show that in the one-loop computation of the fermion self-energy $\sum(k, i\nu_n)$ only one contribution coming from the three-leg vertex U_a is significant. Because of the form of the free-boson propagator (2.4) the part coming from the four-leg vertex U_{ab} vanishes.

Therefore the self-energy $\sum(k, i\nu_n)$ is given by

$$\begin{aligned}\sum(k, i\nu_n) &= \frac{1}{N_s}\sum_{\omega,q} U_a\, D^{ba}_{(0)}(\omega, q)\, U_b\, G(\nu + \omega, k + q)\\ &= \sum_q (f(k, q) + \omega\, g(k, q)) \sum_\omega \frac{G(\nu + \omega, k + q)}{\omega^2 + \omega_q^2},\end{aligned} \tag{3.2}$$

where

$$f(k, q) = \frac{J'z(1+\rho)^2}{4N_s}\left(\varepsilon_k^2 + \varepsilon_{k'}^2 - 2\gamma_q\, \varepsilon_k\, \varepsilon_{k'}\right), \tag{3.3}$$

$$g(k, q) = \frac{-2is(1+\rho)}{(s + s')N_s}\left(\varepsilon_{k'}^2 - \varepsilon_k^2\right). \tag{3.4}$$

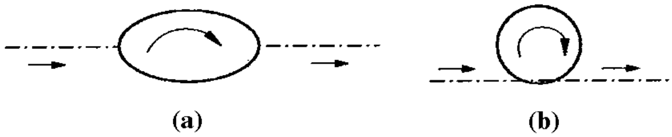

Fig. 1.

By using standard techniques, the following expression for the fermionic self-energy at zero temperature is found

$$\sum(k, i\nu_n) = \frac{(1+\rho)}{2N_s} t^2 z^2 \sum_q \times \left[\frac{(sign\,\gamma_q)\,\gamma_k \sqrt{\left[1 - \sqrt{(1-\gamma_q^2)}\right]} - \gamma_{k+q}\sqrt{\left[1 + \sqrt{(1-\gamma_q^2)}\right]}}{\sqrt{(1-\gamma_q^2)}} \right]^2 \times \frac{s(s+s')(1+\rho)}{i\nu_n - \omega_q - \mu - \sum(k+q, i\nu_n - \omega_q)}, \tag{3.5}$$

where the relation $\varepsilon_k = -z\,t\,\gamma_k$ was used.

Now by using Eq. (2.11), the Eq. (3.2) takes the final form

$$\sum(k, i\nu_n) = \frac{(1+\rho)}{2N_s} t^2 z^2 \sum_q \frac{(u_q\,\gamma_{k+q} + v_q\,\gamma_k)^2}{i\nu_n - \omega_q - \mu - \sum(k+q, i\nu_n - \omega_q)}. \tag{3.6}$$

The expression (3.6) is useful in the strong coupling case ($t > J$). Moreover, in order to describe a metallic phase where the holes move coherently on the lattice, it is necessary to solve the self-consistent Eq. (3.6), which must be carried out numerically.

Once an appropriate self-energy function $\sum(k, i\nu_n)$ is found, the propagator $G(k, \nu)$ remains well defined, and so it is possible to compute numerically the spectral function $B(k, \nu)$.

It can be seen that the Eq. (3.6) is the generalization for finite values of holes to the equivalent equation coming from the spin-polaron theories (see Martinez and Horsch, 1991; Schmitt-Rink *et al.*, 1988). In fact this is a strong proof of the

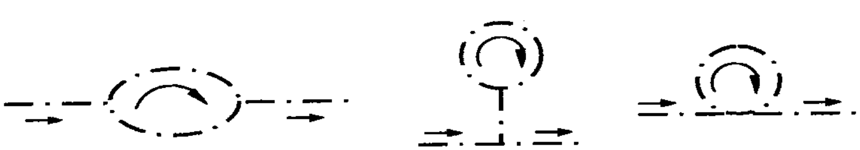

Fig. 2.

correctness of the quantum procedure developed in the t–J Lagrangian model.

On the other hand, the renormalized spin-wave propagators can be obtained by means of the Dyson equation $(\mathcal{D})^{-1}_{ab} = (\mathcal{D})^{-1}_{(0)ab} - \prod_{ab}$. By looking at the diagrammatics it can be seen that the boson self-energy $\prod_{ab}$ is given by the sum of the contributions of all the following one-loop diagrams

$$\prod_{ab}^{(1)}(\omega, q) = (-1)\frac{1}{N_{\mathrm{s}}}\sum_{\nu,k} U_a\, G(k, \nu_n)\, U_b\, G(k-q, \nu_n - \omega_n). \tag{3.7a}$$

$$\prod_{ab}^{(2)}(\omega, q) = (-1)\frac{1}{2N_{\mathrm{s}}}\sum_{\nu,k} U_{ab}\, G(k, \nu_n), \tag{3.7b}$$

and

$$\prod_{ab}^{(3)}(\omega, q) = \frac{1}{2N_{\mathrm{s}}}\sum_{\omega',q'} F_{adc}\,(\omega, \omega')\, D^{de}_{(0)}(\omega', q')\, F_{ebf}\,(\omega, \omega') \times D^{fc}_{(0)}(\omega' - \omega, q' - q). \tag{3.7c}$$

$$\prod_{ab}^{(4)}(\omega, q) = \frac{1}{2N_{\mathrm{s}}}\sum_{\omega',q'} F_{acb}\,(\omega)\, D^{cd}_{(0)}\,(0)\, F_{def}\,(\omega')\, D^{ef}_{(0)}(\omega', q'). \tag{3.7d}$$

$$\prod_{ab}^{(5)}(\omega, q) = \frac{1}{2N_{\mathrm{s}}}\sum_{\omega',q'} F_{acdb}\,(\omega, \omega')\, D^{cd}_{(0)}\,(\omega', q'). \tag{3.7e}$$

where N_{s} is the lattice number of sites, and the symmetry factors have been taken into account. In the above diagrams the line represents fermions and the doted and dashed line represents bosons.

In the antiferromagnetic configuration the diagrams containing one fermionic loop complicate the boson self-energy expression. In this case the associated matrix $\prod_{ab}(q, \omega_n)$ contains contributions to the $\prod_{+3}(q, \omega_n)$ and $\prod_{-3}(q, \omega_n)$ components, given rise to longitudinal spin-wave modes. Of course, in the regime we are working, these contributions are small corrections to the transversal spin-wave components. But it is important to note that the Lagrangian model (2.3) takes into account all the possible interactions without any approximation.

With the aim to confront our results with those obtained by means of the t–J Hamiltonian model in the slave-fermion Schwinger boson representation we only retain terms of order t^2. These terms coming from the diagram of Fig. 1(a) produce the renormalization in the transversal spin wave modes given rise to a significant softening of the spin excitations.

The small corrections produced by the longitudinal contributions of the diagrams of Fig. 1(b), as well as the contributions coming from the diagrams containing pure bosonic vertices of Fig. 2, will be evaluated in a future paper.

4. MAGNETIC PROPERTIES IN ANTIFERROMAGNETS WEAKLY DOPED

As was mentioned above the propagation of a single hole in a two dimensional antiferromagnets has been studied in the framework of the t–J Hamiltonian model in a Schwinger boson representation using the (SCBA) (see for instance Kane *et al.*, 1990; Liu and Manousakis, 1992; Martinez and Horsch, 1991; Pimentel *et al.*, 1999; Schmitt-Rink *et al.*, 1988). Later on, this approach was extended to the case of finite concentration of holes (see Igarashi and Fulde, 1992; Izyumov, 1997; Kyung and Mukhin, 1997; Plakida *et al.*, 1994). In both situations when the density of holes is small, it is well known that several results are essentially the same.

In particular, because of the strong coupling between holes and spin excitations a hole propagates coherently having a quasiparticle band-width $\sim J$ and energy minima at momenta $k_i = (\pm\pi/2, \pm\pi/2)$. When the spectral density is computed it shows a cuasiparticle peak of intensity $\sim(J/t)^{2/3}$, and a broad incoherent multiple spin-wave continuum located at higher energies of width $\sim 2zt$. Now, from our model we must study the effect of the coherent and the incoherent motion of holes interacting with spin wave by computing the softening and damping of the spin excitations.

When the (SCBA) approach is used we can suppose that the hole spectral function $B(k, \nu)$ is composed by a coherent part corresponding to the quasiparticle peak plus an incoherent continuum. So, the spectral function takes the approximate form (Pimentel *et al.*, 1999)

$$B(k, \nu) = [B_{\text{coh}}(k, \nu) + B_{\text{inc}}(k, \nu)]\mathcal{F}^{\pm}(k)\theta(\pm\nu) \tag{4.1}$$

where the Fermi surface $\mathcal{F}^{-}(k) = \sum_{i=1}^{4} \theta(k_{\text{F}} - |k - k_i|)$, $\mathcal{F}_k^{+}(k) = 1 - \mathcal{F}^{-}(k)$, and k_{F} is the Fermi momentum.

In Eq. (4.1) the coherent and incoherent parts respectively read

$$B_{\text{coh}}(k, \nu) = Z_k \delta(\nu - \varepsilon_k + \mu), \tag{4.2a}$$

$$B_{\text{inc}}(k, \nu) = h\theta(|\nu| - zJ/2)\theta(2zt + zJ/2 - |\nu|). \tag{4.2b}$$

where Z_k is the intensity of the quasiparticle state, and so the height of the continuum region is $\sim(1 - Z_k)/2zt$ satisfiying the sum rule $\int B(k, \nu)d\nu = 1$. The energies are measured with respect to the Fermi level, and near the minima at k_i the quasiparticle dispersion in Eq. (4.2a) is written $\varepsilon_k = \varepsilon_{\min} + (k - k_i)^2/2m$ with an effective mass m.

In order to determine the renormalized spin-wave energy ω_q we only take into account contributions of order t^2 that are those coming from the self-energy $\prod_{ab}(q, \omega)$ defined in Eq. (3.9a). Moreover, from this equation and the expresion (4.1) it can be seen that the self-energy $\prod_{ab}(q, \omega)$ presents three contributions $\prod_{ab}(q, \omega)_{\text{coh,coh}}$, $\prod_{ab}(q, \omega)_{\text{coh,inc}}$and $\prod_{ab}(q, \omega)_{\text{inc,inc}}$.

The dressed hole propagator (3.1) in terms of the spectral function is written

$$G(k, i\nu_n) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} d\nu' \frac{B(k, \nu')}{i\nu_n - \nu'} \tag{4.3}$$

and so the four components of the self-energy $\prod_{ab}(a, b = \pm)$ of Fig. 1(a) diagram reads

$$\prod_{ab} = (-1)\frac{1}{N_s} \sum_{k,\nu_n} U_a(k, q, \nu_n\omega_n)U_b(k, q, \nu_n\omega_n) \int_{-\infty}^{+\infty} \frac{d\nu'}{(2\pi)} \int_{-\infty}^{+\infty} \frac{d\nu''}{(2\pi)} \times \frac{B(k, \nu')}{i\nu_n - \nu'} \frac{B(k - q, \nu'')}{i(\nu_n - \omega_n) - \nu''} \tag{4.4}$$

The self-energy (4.4) can be computed straightforward by using the expression (4.1) for the spectral function. Once this is done and the components of the dressed propagator $D_{ab}(q, \omega)$ are found, the renormalized spin-wave energy ω_q is determined by the poles of such propagator, i.e, by the condition

$$\left[\left(D_0^{-+}\right)^{-1} - \prod^{+-}\right]\left[\left(D_0^{+-}\right)^{-1} - \prod^{-+}\right] - \prod^{++}\prod^{--} = 0. \tag{4.5}$$

As well known, by analizing the region where $\mathbf{Im}\prod(q, \omega) = 0$, it is possible to obtain the softening in the frequency ω_q of the antiferromagnetic magnon. For the renormalized frequency we found the following expression:

$$\omega_q^R = \omega_q + \mathbf{Re}\prod^{+-} = \omega_q[1 - r(q)], \tag{4.6}$$

where the three parts coherent–coherent, coherent–incoherent, and incoherent–incoherent of the self-energy $\prod^{+-}$ give contributions to the function $r(q)$. Of course the function $r(q)$ defined in Eq. (4.6) in two dimension is the same to that written in Pimentel *et al.*, (1999).

Analogously, in order to evaluate the damping in the spin excitations we must study the region where $\mathbf{Im}\prod(q, \omega) \neq 0$, obtaining an inverse lifetime given by

$$\Gamma_q = -2\mathbf{Im}\prod^{+-}(q, \omega_q), \tag{4.7}$$

where the contribution to the damping is determined only by the coherent motion of holes, i.e, $\mathbf{Im}\prod_{coh,coh}^{+-}$.

Moreover, as well known for finite values of the hole density ρ, the renormalization factor $Z_c < 1$ giving rise to a reduction of the spin-wave velocity.

Looking at the Hamiltonian t–J model in the Schwinger boson representation (Pimentel *et al.*, 1999), we can conclude that the renormalization of the magnetic properties produced by the interaction vertex of this model, is precisely the same renormalization produced by the Lagrangian diagrammatics by taking into account only the terms proportional to t^2.

As was said above to compute the renormalization effects, only strong contributions of order t^2 have been considered, because in the present regime all the remaining contributions represent small corrections. The Lagrangian model includes other small corrections, particularly longitudinal contributions which are not present in the Hamiltonian t–J model.

So, under this conditions we can conclude that the Lagrangian model and the Hamiltonian t–J model in the Schwinger boson representation gives the same results for the softening and damping of the spin excitations.

The same conclusion is valid for the transverse spin susceptibility $\chi_{\perp} = \chi_{\perp}(q = 0, \omega = 0)$ which in terms of the renormalized spin-wave propagator is given by $\chi_{\perp} = -\lim_{q\to 0}\left[\frac{(1-\gamma_q)}{(1+\gamma_q)}\right]^{1/2}[Re\, D^{+-}(q, 0) + Re\, D^{++}(q, 0)]$. Finally, from the above expression the transverse spin susceptibility is written in terms of self-energy components as follows

$$\chi_{\perp} = \lim_{q\to 0}\frac{1}{zJ(1+\gamma_q)}\left[1 - \frac{2}{zJ\left(1-\gamma_q^2\right)^{1/2}}\left[Re\prod^{+-}(q,0) + Re\prod^{++}(q,0)\right]\right], \tag{4.8}$$

where in the limit $q \to 0$ only the coherent parts of the self-energy components gives contributions. The increasing of the transverse susceptibility is because from the calculations results a renormalization factor $Z_{\chi} > 1$.

The above results constitute a strong test about the correctness of our Lagrangian formalism.

5. CONCLUSIONS

The first-order Lagrangian formalism for the t–J model proposed and developed in the context of the path-integral formalism (Foussats *et al.*, 2000 a,b, 2002), has been checked by analyzing and computing several magnetic properties in ferromagnets (see Foussats *et al.*, 2002), as well as in antiferromagnets.

The remarkable feature of our approach is that the Hubbard X-operators are used as field variables allowing to describe without any decoupling assumption, spin and charge fluctuations on the atomic lattice site.

As it was showed in Foussats *et al.* (2002) (Eq. (6.21) for ferromagnets, our approach gives response to the thermal softening of the ferromagnetic magnon frequency, and the expression we found is the generalization for different from zero hole density to that obtained by means of the non-linear spin wave model (Mattis, 1981).

It is important to note that our model accounts for the softening effect when only oneloop computations—without any vertex correction—is considered. We think this fact is important because it simplifies the computation. As it can be seen, in the framework of nonlinear spin-wave model, the softening of the magnon frequency is obtained by including vertex corrections.

As it was shown in the present paper our diagrammatics accounts correctly for the renormalization of magnetic properties for weakly doped antiferromagnets. The results are those obtained by means of the t–J Hamiltonian model in a Schwinger boson representation, when only contributions of t^2 order (transversal components) are taking into account. Besides, the diagrammatics of the Lagrangian model contains pure spin-wave interaction vertices as well as different from zero longitudinal components in the self-energy giving rise to small corrections in the dressed spin-wave propagator which can be appropriately computed.

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